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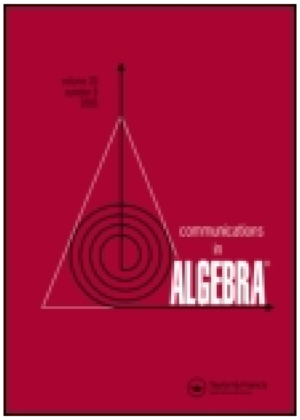

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CODIMENSION GROWTH AND NON-KOSZULITY OF NOVIKOV OPERAD

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An algebra with identities $a \circ (b \circ c - c \circ b) = (a \circ b) \circ c - (a \circ c) \circ b$ and $a \circ (b \circ c) = b \circ (a \circ c)$ is called Novikov. We construct free Novikov base in terms of Young diagrams. We show that codimensions exponent for a variety of Novikov algebras exists and is equal to 4. We prove that Novikov operad is not Koszul.

Key Words: Codimensions sequence; Growth of a variety; Koszulity of operad; Novikov algebra; Operad; Polynomial identities.

2000 Mathematics Subject Classification: Primary 16R10, 17A50, 17A30, 17D25, 17C50.

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1. INTRODUCTION

A variety of algebras is a class of algebras satisfying some polynomial identities. One of important parameters of varieties is so-called a codimension growth. If $\mathcal{V}$ is a variety and $N_n(\mathcal{V})$ is a multilinear part of its free algebra generated by n elements, then $c_n(\mathcal{V}) = dim\, N_n(\mathcal{V})$ is called the nth codimension of $\mathcal{V}$. Codimension growth is defined by a sequence of codimensions $c_1, c_2, c_3, \ldots$. The *codimension exponent* is defined as

$$Exp(\mathcal{V}) = \lim c_n(\mathcal{V})^{1/n}.$$

Natural questions appear whether this exponent exists and whether it is an integer. In an associative case, these questions are well studied. It was proved that $Exp(\mathcal{V})$ exists and it is an integer for any proper variety of associative algebras ([5]). Constructions of free bases for Lie algebras are well known (about Hall–Lyndon–Shrishov bases see, for example, [7]).

In this article we consider a class of non-associative algebras. An algebra $A = (A, \circ)$ is called a right-*Novikov* ([1, 3, 6]), if it satisfies the identities

$$(a, b, c) = (a, c, b)$$
$$a \circ (b \circ c) = b \circ (a \circ c),$$

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for any $a, b, c \in A$. Here

$$(a, b, c) = a \circ (b \circ c) - (a \circ b) \circ c$$

is an associator. Similarly, left-*Novikov* algebras are defined by identities

$$(a, b, c) = (b, a, c)$$
$$(a \circ b) \circ c = (a \circ c) \circ b,$$

for any $a, b, c \in A$. Note that any right-(left-)Novikov algebra under opposite multiplication $(a, b) \mapsto a \circ_{op} b = b \circ a$ becomes left-(right-)Novikov. Novikov algebras are Lie-admissible.

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Example. $A = \mathbf{C}[x]$ under multiplication $a \circ b = \partial(a)b$ is right-Novikov.

If not stated otherwise, by Novikov algebras we will mean right-Novikov algebras. Let *Nov* be a variety of Novikov algebras and N_n be a multilinear part of free Novikov algebra generated by n elements. Let

$$Exp(Nov) = \lim_{n\to\infty} (\dim N_n)^{1/n}$$

be codimensions growth of Novikov variety.

We give construction of free Novikov base in terms of Young diagrams and use this base for calculation of generating function and codimension growth. We prove that Novikov operad is not Koszul. Our main results are the following theorem.

Theorem 1.1. *Codimensions sequence for Novikov varety is given by*

$$\dim N_n = \binom{2n-2}{n-1}.$$

Codimension exponent of Novikov variety exists and

$$Exp(Nov) = 4.$$

Generating function of codimensions sequence $\sum_{i\geq 0} N_i x^i$ *is equal to* $x(1-4x)^{-1/2}$.

Theorem 1.2. *Dual operad to left-(right-)Novikov operad is right-(left)-Novikov. Novikov operad is not Koszul.*

2. FREE BASE FOR NOVIKOV ALGEBRAS

In [2] are given constructions of a base of free Novikov algebra in terms of r-elements and in terms of rooted trees. In this section, we give construction of free base in terms of Young diagrams.

Recall that a Young diagram is a set of boxes (we denote them by bullets) with non-increasing numbers of boxes in each row. Rows and columns are numbered from top to bottom and from left to right. Let k be the number of rows and r_i be the number of boxes in the ith row. The total number of boxes, $r_1 + \cdots + r_k$, is called *degree* of Young diagram.

To construct Novikov diagram, we need to complement Young diagram by one box, we call it as "a nose". Namely, we need to add the first row by one more box,

$$\begin{array}{cccc} \bullet & \cdots & \bullet & \bullet & \bullet \\ \bullet & \cdots & \bullet & \bullet & \\ \vdots & \cdots & \vdots & \vdots & \\ \bullet & \cdots & \bullet & & \end{array} \quad \mapsto \quad \begin{array}{cccccc} \bullet & \cdots & \bullet & \bullet & \bullet & \circ \\ \bullet & \cdots & \bullet & \bullet & & \\ \vdots & \cdots & \vdots & \vdots & & \\ \bullet & \cdots & \bullet & & & \end{array} .$$

The number of boxes in Novikov diagram is called its *degree*. So, difference between degrees of Novikov diagram and corresponding Young diagram is equal to 1.

Let us given an alphabet (ordered set) Ω. To construct Novikov tableau on Ω, we need to feel Novikov diagrams by elements of Ω. Denote by $a_{i,j}$ an element of Ω in the box (i, j), that is, a cross of ith row by jth column. The feeling rule is the following:

a) $a_{i,1} \geq a_{i+1,1}$, if $r_i = r_{i+1}$, $i = 1, 2, \ldots, k-1$;
b) The sequence $a_{k,2} \cdots a_{k,r_k} a_{k-1,2} \cdots a_{k-1,r_{k-1}} \cdots a_{1,2} \cdots a_{1,r_1} a_{1,r_1+1}$ is nondecreasing.

In particular, all boxes beginning from the second place in each row are labeled by nondecreasing elements of the alphabet. Denote by R_n a set of Novikov tableaux labeled by Ω with $n = r_k + \cdots + r_1 + 1$ boxes.

Let $F(\Omega)$ be free Novikov algebra generated by Ω. Let $F_n(\Omega)$ be its subspace generated by basic elements of degree n. Correspond to any Novikov tableaux

$$\begin{array}{cccccc} a_{1,1} & \cdots & \cdots & a_{1,r_1-1} & a_{1,r_1} & a_{1,r_1+1} \\ a_{2,1} & \cdots & a_{2,r_2-1} & a_{2,r_2} & & \\ \vdots & \cdots & \vdots & \vdots & & \\ a_{k,1} & \cdots & a_{k,r_k} & & & \end{array}$$

an element

$$X = X_k \circ (X_{k-1} \circ (\cdots \circ (X_2 \circ X_1) \cdots)),$$

(right-normed bracketing) where

$$X_i = (\cdots((a_{i,1} \circ a_{i,2}) \circ a_{i,3}) \cdots \circ a_{i,r_i-1}) \circ a_{i,r_i}, \qquad 1 < i \leq k,$$
$$X_1 = (\cdots((a_{1,1} \circ a_{1,2}) \circ a_{1,3}) \cdots \circ a_{1,r_1}) \circ a_{1,r_1+1}$$

(left-normed bracketing). All base elements of free Novikov algebra $F(\Omega)$ are obtained by this way. In particular, $\dim F_n(\Omega) = |R_n|$.

As an example, let us construct base elements of polylinear part N_4 of free Novikov algebra generated by 4 elements a, b, c, d.

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Young diagrams of degree 3:

$$
\begin{array}{l}
\bullet \\
\bullet \\
\bullet
\end{array}
\qquad
\begin{array}{ll}
\bullet & \bullet \\
\bullet &
\end{array}
\qquad
\begin{array}{lll}
\bullet & \bullet & \bullet
\end{array}
$$

Novikov diagrams of degree 4:

$$
\begin{array}{ll}
\bullet & \circ \\
\bullet & \\
\bullet &
\end{array}
\qquad
\begin{array}{lll}
\bullet & \bullet & \circ \\
\bullet & &
\end{array}
\qquad
\begin{array}{llll}
\bullet & \bullet & \bullet & \circ
\end{array}
$$

Novikov tableaux of degree 4 generated by elements a, b, c, d:

$$
\begin{array}{ll}
c & d \\
b & \\
a &
\end{array}
\quad
\begin{array}{ll}
d & c \\
b & \\
a &
\end{array}
\quad
\begin{array}{ll}
d & b \\
c & \\
a &
\end{array}
\quad
\begin{array}{ll}
d & a \\
c & \\
b &
\end{array}
$$

$$
\begin{array}{lll}
b & c & d \\
a & &
\end{array}
\quad
\begin{array}{lll}
c & b & d \\
a & &
\end{array}
\quad
\begin{array}{lll}
d & b & c \\
a & &
\end{array}
$$

$$
\begin{array}{lll}
a & c & d \\
b & &
\end{array}
\quad
\begin{array}{lll}
c & a & d \\
b & &
\end{array}
\quad
\begin{array}{lll}
d & a & c \\
b & &
\end{array}
$$

$$
\begin{array}{lll}
a & b & d \\
c & &
\end{array}
\quad
\begin{array}{lll}
b & a & d \\
c & &
\end{array}
\quad
\begin{array}{lll}
d & a & b \\
c & &
\end{array}
$$

$$
\begin{array}{lll}
a & b & c \\
d & &
\end{array}
\quad
\begin{array}{lll}
b & a & c \\
d & &
\end{array}
\quad
\begin{array}{lll}
c & a & b \\
d & &
\end{array}
$$

$$
a\ b\ c\ d \qquad b\ a\ c\ d \qquad c\ a\ b\ d \qquad d\ a\ b\ c
$$

So, multilinear part of free Novikov algebra in degree 4 is 20-dimensional, and the following elements form the base:

$$
\begin{array}{llll}
a \circ (b \circ (c \circ d)), & a \circ (b \circ (d \circ c)), & a \circ (c \circ (d \circ b)), & b \circ (c \circ (d \circ a)), \\
a \circ ((b \circ c) \circ d), & a \circ ((c \circ b) \circ d), & a \circ ((d \circ b) \circ c), & \\
b \circ ((a \circ c) \circ d), & b \circ ((c \circ a) \circ d), & b \circ ((d \circ a) \circ c), & \\
c \circ ((a \circ b) \circ d), & c \circ ((b \circ a) \circ d), & c \circ ((d \circ a) \circ b), & \\
d \circ ((a \circ b) \circ c), & d \circ ((b \circ a) \circ c), & d \circ ((c \circ a) \circ b), & \\
((a \circ b) \circ c) \circ d, & ((b \circ a) \circ c) \circ d, & ((c \circ a) \circ b) \circ d, & ((d \circ a) \circ b) \circ c.
\end{array}
$$

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3. CODIMENSIONS GROWTH OF NOVIKOV VARIETY

Let $\lambda = 1^{m_1(\lambda)} 2^{m_2(\lambda)} \cdots$ be a partition of $n-1$, i.e.,

$$|\lambda| = \sum_{i \geq 1} i\, m_i(\lambda) = n - 1.$$

Let

$$m(\lambda) = \sum_{i \geq 1} m_i(\lambda).$$

For $m, m_1, m_2, \ldots, m_n$, such that $m = m_1 + m_2 + \cdots + m_n$, let

$$\binom{m}{m_1, m_2, \ldots, m_n} = \frac{m!}{m_1! m_2! \cdots m_n!}$$

be a multinomial coefficient.

Lemma 3.1.

$$\sum_{|\lambda| = n-1} \binom{m(\lambda)}{m_1(\lambda), m_2(\lambda), \ldots} \binom{n}{m(\lambda)} = \binom{2(n-1)}{n-1}.$$

Proof. By Vandermonde convolution relation ([8], Chapter 1.3, formulae (3a)),

$$\binom{n+p}{m} = \sum_{s \geq 0} \binom{n}{m-s} \binom{p}{s}.$$

By ([8] Chapter 4.5, formulae (21)), for fixed n and m, the following relation takes place:

$$\sum_{m_1, m_2, \ldots} \binom{m}{m_1, m_2, \ldots, m_n} = \binom{n-1}{m-1},$$

where summation is over $m_1, m_2, \ldots, m_n$, such that $m = m_1 + m_2 + \cdots + m_n$, $n = m_1 + 2m_2 + \cdots + nm_n$.

By these relations,

$$\begin{aligned}
\sum_{|\lambda| = n-1} \binom{m(\lambda)}{m_1(\lambda), m_2(\lambda), \ldots} \binom{n}{m(\lambda)} &= \sum_{s \geq 1} \binom{n}{s} \sum_{|\lambda| = n-1, m(\lambda) = s} \binom{s}{m_1(\lambda), m_2(\lambda), \ldots} \\
&= \sum_{s \geq 1} \binom{n}{s} \binom{n-2}{s-1} = \sum_{s \geq 1} \binom{n}{s} \binom{n-2}{n-s-1} \\
&= \binom{2(n-1)}{n-1}.
\end{aligned}$$

Lemma 3.2.

$$\lim_{n\to\infty}\binom{2n-2}{n-1}^{1/n}=4.$$

Proof. For $a\in\mathbf{Z}$, denote by $a!!$ a product of positive integers $a, a-2, a-4$, and so on. For example, $(2n-1)!!$ is a product of odd numbers between 1 and $2n-1$. We have

$$(2n-4)!!\le(2n-3)!!\le(2n-2)!!.$$

Thus

$$\frac{2^{n-2}}{n-1}\le\frac{(2n-3)!!}{(n-1)!}\le 2^{n-1}.$$

Since

$$\binom{2(n-1)}{n-1}=\frac{2(n-1)!}{((n-1)!)^2}=\frac{2^{n-1}(2n-3)!!}{(n-1)!},$$

we have

$$\frac{2^{2n-3}}{n-1}\le\binom{2(n-1)}{n-1}\le 2^{2(n-1)}.$$

It remains to note that

$$\lim_{n\to\infty}\left(\frac{2^{2n-3}}{(n-1)}\right)^{1/n}=\lim_{n\to\infty}(2^{4n-2})^{1/n}=4.$$

Proof of Theorem 1.1. As we mentioned above, any polylinear base element of free Novikov algebra of degree n corresponds to Young diagram of degree $n-1$. Suppose that it has all together m rows, namely, m_1 rows with i_1 boxes, m_2 rows with i_2 boxes, etc., m_k rows with i_k boxes, where $i_1>i_2>\cdots>i_k$. So, $\sum_{s=1}^{k} i_s m_s=n-1$, and such Young diagram looks like the following:

$$\begin{array}{l}
m_1\left\{\begin{array}{cccccc}\bullet&\cdots&\bullet&\bullet&\bullet&\bullet\\ \vdots&\cdots&\vdots&\vdots&\vdots&\vdots\\ \bullet&\cdots&\bullet&\bullet&\bullet&\bullet\end{array}\right.\\
m_2\left\{\begin{array}{cccc}\bullet&\cdots&\bullet&\bullet\\ \vdots&\cdots&\vdots&\vdots\\ \bullet&\cdots&\bullet&\bullet\\ \vdots&\cdots&\vdots&\end{array}\right.\\
m_k\left\{\begin{array}{ccc}\bullet&\cdots&\bullet\\ \vdots&\cdots&\vdots\\ \bullet&\cdots&\bullet.\end{array}\right.
\end{array}$$

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Set $m_i = 0$, if $i > k$. The Novikov diagram corresponding to such Young diagram, filled by n different letters, is uniquiely defined by its first column. The first column can be choosen in

$$\binom{n}{m_1, m_2, \ldots, m_n} = \binom{m}{m_1, m_2, \ldots, m_n}\binom{n}{m}$$

ways. Therefore, by Lemma 3.1

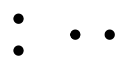
$$\dim R_n = \sum \binom{m}{m_1, m_2, \ldots, m_n}\binom{n}{m} = \binom{2(n-1)}{n-1}$$

(summation is over $m_1, m_2, \ldots, m_n$ such that $\sum_s i_s m_s = n - 1$.) By Lemma 3.2, $Exp(Nov) = 4$. The remainder statements of Theorem 1.1 are evident.

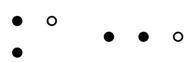
4. NON-KOSZULNESS OF NOVIKOV OPERAD

There are 12 multilinear elements of degree 3. But only 6 of them form base. Let us construct multilinear base elements of degree 3 for free Novikov algebra. We will follow instructions given in Section 2.

Young diagrams of degree 2:



Novikov diagrams of degree 3:



Novikov tableaux of degree 3 generated by elements a, b, c:

$$\begin{array}{ll} b & c \\ a & \end{array} \quad \begin{array}{ll} c & b \\ a & \end{array} \quad \begin{array}{ll} c & a \\ b & \end{array} \quad a\ b\ c \quad b\ a\ c \quad c\ a\ b$$

Multilinear base elements of degree 3:

$$a \circ (b \circ c), \quad a \circ (c \circ b), \quad c \circ (a \circ b), \quad (a \circ b) \circ c, \quad (b \circ a) \circ c, \quad (c \circ a) \circ b.$$

Below we give presentation of 6 non-base elements of degree 3 as a linear combination of above constructed base elements of degree 3:

$$b \circ (a \circ c) = a \circ (b \circ c), \quad c \circ (a \circ b) = a \circ (c \circ b), \quad c \circ (b \circ a) = b \circ (c \circ a),$$

$$(a \circ c) \circ b = (a \circ b) \circ c + a \circ (c \circ b) - a \circ (b \circ c),$$

$$(b \circ c) \circ a = -a \circ (b \circ c) + b \circ (c \circ a) + (b \circ a) \circ c,$$

$$(c \circ b) \circ a = (c \circ a) \circ b - a \circ (c \circ b) + b \circ (c \circ a).$$

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Then

$$
\begin{aligned}
&[[a\otimes u, b\otimes v], c\otimes w]\\
&\quad = (a\circ b)\circ c\otimes (u\cdot v)\cdot w - (b\circ a)\circ c\otimes (v\cdot u)\cdot w\\
&\qquad - c\circ (a\circ b)\otimes w\cdot (u\cdot v) + c\circ (b\circ a)\otimes w\cdot (v\cdot u)\\
&\quad = (a\circ b)\circ c\otimes (u\cdot v)\cdot w - (b\circ a)\circ c\otimes (v\cdot u)\cdot w\\
&\qquad - a\circ (c\circ b)\otimes w\cdot (u\cdot v) + b\circ (c\circ a)\otimes w\cdot (v\cdot u)\\
&\quad = ((a\circ b)\circ c)\otimes ((u\cdot v)\cdot w) + ((b\circ a)\circ c)\otimes (-(v\cdot u)\cdot w)\\
&\qquad + (a\circ (c\circ b))\otimes (-w\cdot (u\cdot v)) + (b\circ (c\circ a))\otimes (w\cdot (v\cdot u)).
\end{aligned}
$$

Similarly,

$$
\begin{aligned}
&[[b\otimes v, c\otimes w], a\otimes u]\\
&\quad = (-a\circ (b\circ c) + b\circ (c\circ a) + (b\circ a)\circ c)\otimes ((v\cdot w)\cdot u)\\
&\qquad + (-(c\circ a)\circ b + a\circ (c\circ b) - b\circ (c\circ a))\otimes ((w\cdot v)\cdot u)\\
&\qquad - (a\circ (b\circ c))\otimes (u\cdot (v\cdot w)) + (a\circ (c\circ b))\otimes (u\cdot (w\cdot v))\\
&\quad = (a\circ (b\circ c))\otimes (-(v\cdot w)\cdot u)\\
&\qquad + (b\circ (c\circ a))\otimes ((v\cdot w)\cdot u) + ((b\circ a)\circ c)\otimes ((v\cdot w)\cdot u)\\
&\qquad + (-(c\circ a)\circ b + a\circ (c\circ b) - b\circ (c\circ a))\otimes ((w\cdot v)\cdot u)\\
&\qquad - (a\circ (b\circ c))\otimes (u\cdot (v\cdot w)) + (a\circ (c\circ b))\otimes (u\cdot (w\cdot v)).
\end{aligned}
$$

Further,

$$
\begin{aligned}
&[[c\otimes w, a\otimes u], b\otimes v]\\
&\quad = ((c\circ a)\circ b)\otimes ((w\cdot u)\cdot v) + (-(a\circ b)\circ c - a\circ (c\circ b)\\
&\qquad + a\circ (b\circ c))\otimes ((u\cdot w)\cdot v) + (b\circ (c\circ a))\otimes (-v\cdot (w\cdot u))\\
&\qquad + (a\circ (b\circ c))\otimes (v\cdot (u\cdot w))\\
&\quad = ((c\circ a)\circ b)\otimes ((w\cdot u)\cdot v) + ((a\circ b)\circ c)\otimes (-(u\cdot w)\cdot v)\\
&\qquad + (a\circ (c\circ b))\otimes (-(u\cdot w)\cdot v) + a\circ (b\circ c))\otimes ((u\cdot w)\cdot v)\\
&\qquad + (b\circ (c\circ a))\otimes (-v\cdot (w\cdot u)) + (a\circ (b\circ c))\otimes (v\cdot (u\cdot w)).
\end{aligned}
$$

Therefore,

$$
\begin{aligned}
&[[a\otimes u, b\otimes v], c\otimes w] + [[b\otimes v, c\otimes w], a\otimes u] + [[c\otimes w, a\otimes u], b\otimes v]\\
&\quad = ((a\circ b)\circ c)\otimes \{(u\cdot v)\cdot w - (u\cdot w)\cdot v\}\\
&\qquad + (a\circ (b\circ c))\otimes \{-(v\cdot w)\cdot u - u\cdot (v\cdot w) + v\cdot (u\cdot w) + (u\cdot w)\cdot v\}\\
&\qquad + (a\circ (c\circ b))\otimes \{-w\cdot (u\cdot v) + (w\cdot v)\cdot u + u\cdot (w\cdot v) - (u\cdot w)\cdot v\}
\end{aligned}
$$

$$+ ((b \circ a) \circ c) \otimes \{-(v \cdot u) \cdot w + (v \cdot w) \cdot u\} + (b \circ (c \circ a)) \otimes$$
$$\{+w \cdot (v \cdot u) + (v \cdot w) \cdot u - (w \cdot v) \cdot u - v \cdot (w \cdot u)\}$$
$$+ ((c \circ a) \circ b) \otimes \{-(w \cdot v) \cdot u + (w \cdot u) \cdot v\}.$$

Thus the Lie-admissibility condition for $A \otimes U$, where A is a free right-Novikov algebra, is equivalent to the following conditions:

$$(u \cdot v) \cdot w - (u \cdot w) \cdot v = 0,$$
$$-(v \cdot w) \cdot u - u \cdot (v \cdot w) + v \cdot (u \cdot w) + (u \cdot w) \cdot v) = 0,$$
$$-w \cdot (u \cdot v) + (w \cdot v) \cdot u + u \cdot (w \cdot v) - (u \cdot w) \cdot v = 0,$$
$$-(v \cdot u) \cdot w + (v \cdot w) \cdot u = 0,$$
$$w \cdot (v \cdot u) + (v \cdot w) \cdot u - (w \cdot v) \cdot u - v \cdot (w \cdot u) = 0,$$
$$-(w \cdot v) \cdot u + (w \cdot u) \cdot v = 0.$$

Note that all of these conditions are consequences of left-symmetric and right-commutative identities. So, $(U, \cdot)$ is left-Novikov, if $(A, \circ)$ is right-Novikov. Similarly, $(U, \cdot)$ is right-Novikov, if $(A, \circ)$ is left-Novikov. These mean that dual operad to right-(left-)Novikov operad is left-(right-)Novikov operad.

We have noted that categories of left-Novikov and right-Novikov algebras are equivalent if we change left-Novikov multiplication to opposite multiplication. In particular, dimensions of multilinear parts of free left-Novikov and free right-Novikov algebras are equal. By Theorem 1.1, these dimensions are $1, 2, 6, 20, 70$ for degrees $1, 2, 3, 4, 5$. Therefore,

$$H(t) = H^!(t) = -t + t^2 - t^3 + 20t^4/24 - 70t^5/120 + O(t^6).$$

Thus,

$$H(H^!(t)) = t + t^5/6 + O(t^6) \neq t.$$

So, by results of [4] left-(right-)Novikov operad is not Koszul. Theorem 1.2 is proved.

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